

Transformasi Fourier

Febrizal, MT

Transformasi Fourier

- Transformasi fourier dari fungsi $f(t)$ dinotasikan dengan $F(\omega)$ dan didefinisikan sbg:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

- Sedangkan invers transformasi fourier fungsi $F(\omega)$ didefinisikan sebagai:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} d\omega$$

- Operasi fourier dinotasikan dengan:

$$F(\omega) = \mathcal{F}[f(t)] \quad \text{and} \quad f(t) = \mathcal{F}^{-1}[F(\omega)]$$

- Contoh:
 - Tentukan transformasi fourier dari fungsi $e^{-at}u(t)$.
- Penyelesaian:

$$F(\omega) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{-1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty}$$

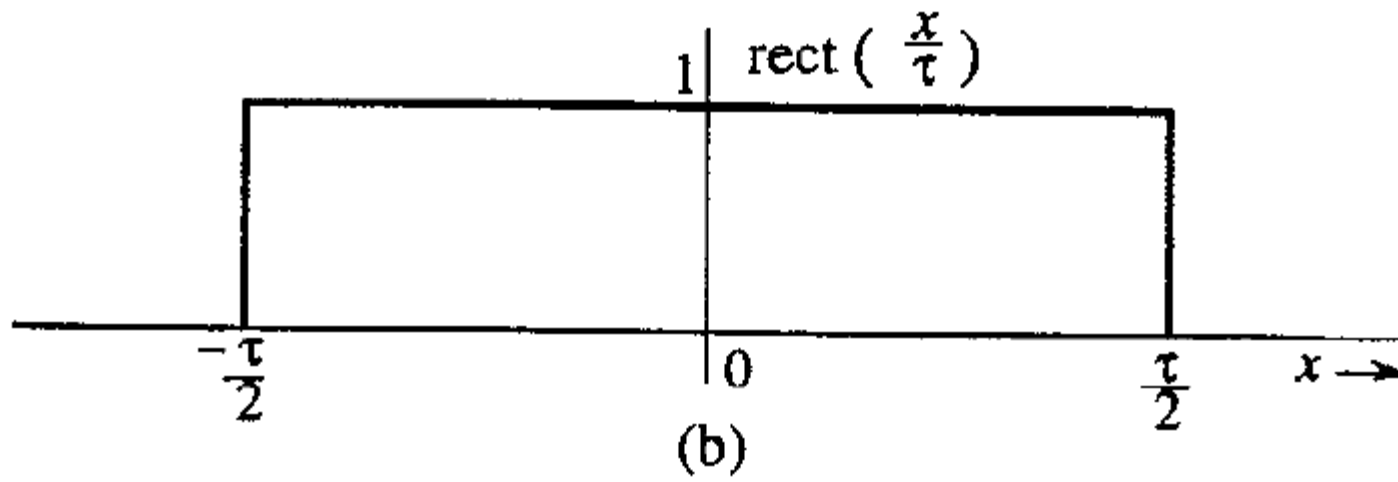
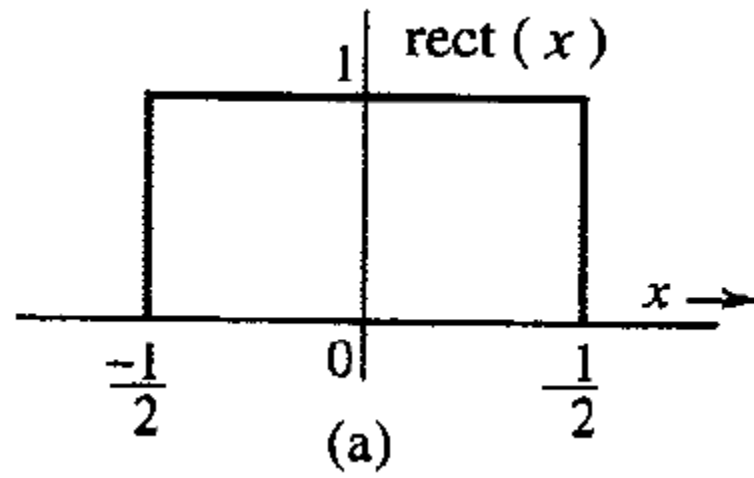
$$F(\omega) = \frac{1}{a+j\omega} \quad a > 0$$

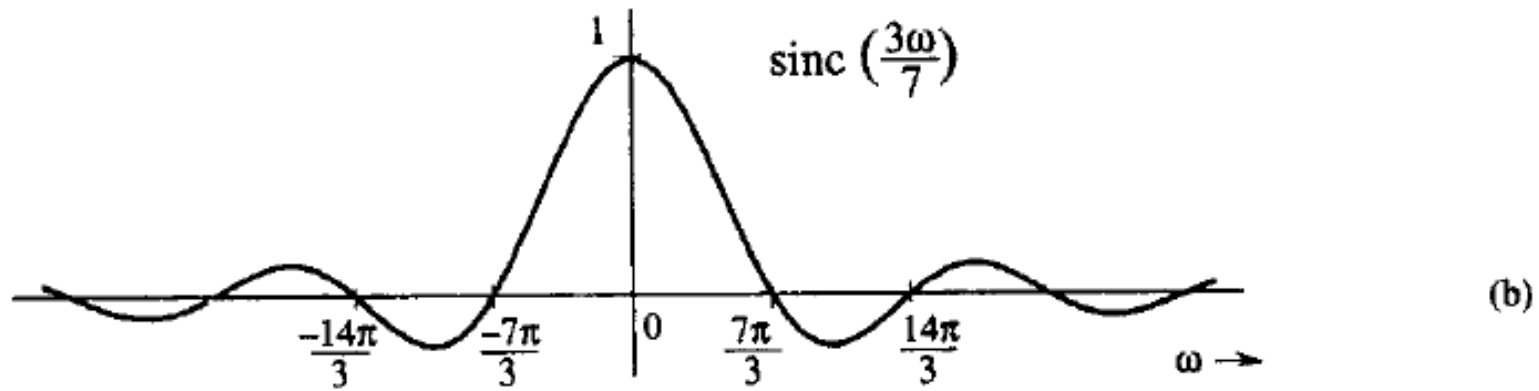
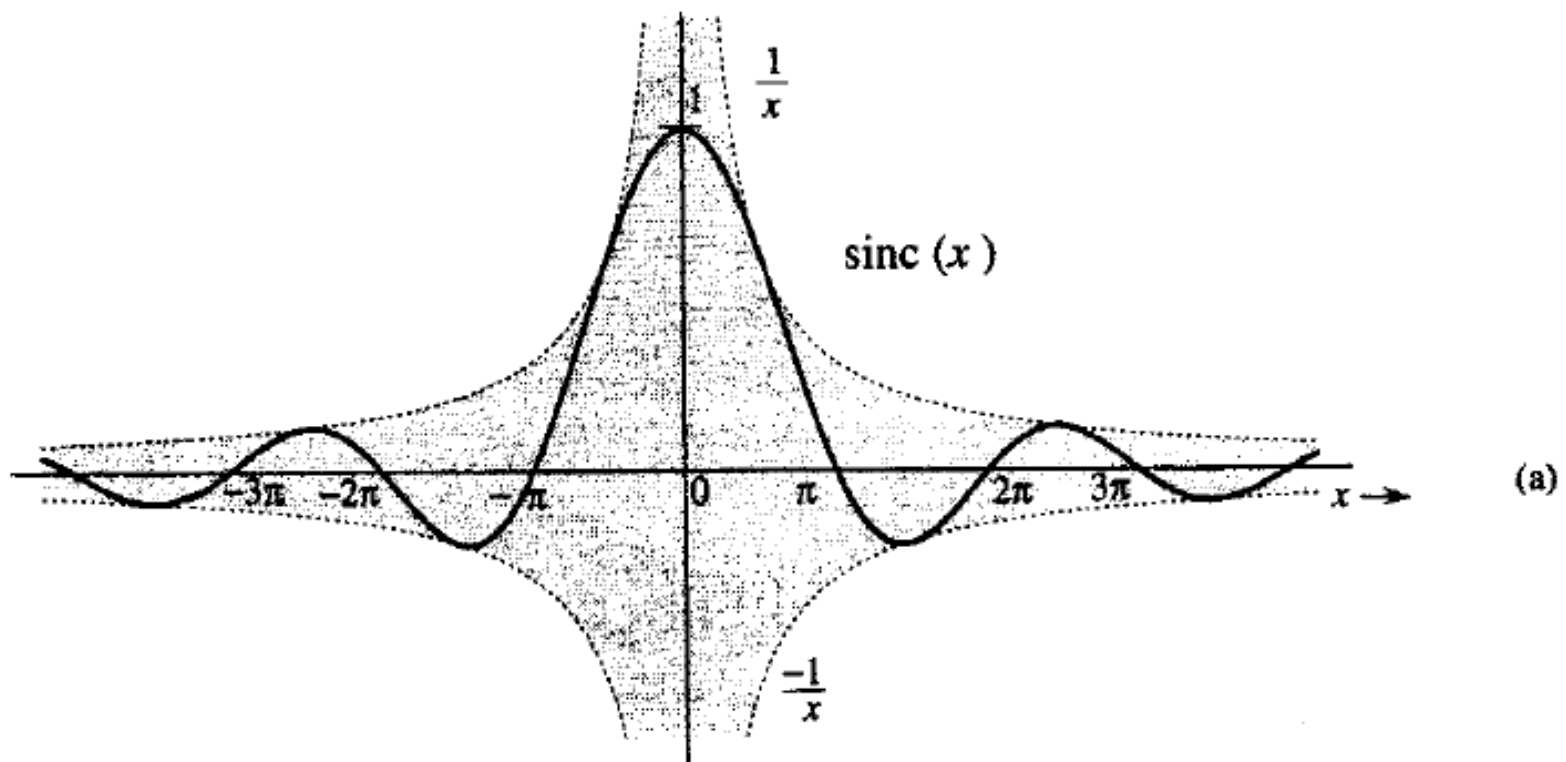
Tabel Transformasi Fourier

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$

6	$\delta(t)$	1
7	1	$2\pi\delta(\omega)$
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
12	$\text{sgn } t$	$\frac{2}{j\omega}$
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$

15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a+j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a+j\omega}{(a+j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\text{rect} \left(\frac{t}{\tau} \right)$	$\tau \text{sinc} \left(\frac{\omega\tau}{2} \right)$	
18	$\frac{W}{\pi} \text{sinc} (Wt)$	$\text{rect} \left(\frac{\omega}{2W} \right)$	
19	$\Delta \left(\frac{t}{\tau} \right)$	$\frac{\tau}{2} \text{sinc}^2 \left(\frac{\omega\tau}{4} \right)$	
20	$\frac{W}{2\pi} \text{sinc}^2 \left(\frac{Wt}{2} \right)$	$\Delta \left(\frac{\omega}{2W} \right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma \sqrt{2\pi} e^{-\sigma^2 \omega^2/2}$	





Sifat² Transformasi Fourier

1. Sifat Simetri

- Sifat ini menyatakan bahwa, jika

$$f(t) \iff F(\omega)$$

- maka

$$F(t) \iff 2\pi f(-\omega)$$

- **Contoh**

$$\underbrace{\text{rect}\left(\frac{t}{\tau}\right)}_{f(t)} \iff \underbrace{\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)}_{F(\omega)}$$

- $F(t)$ sama dengan $F(\omega)$ yang diganti ω dengan t , dan $f(-\omega)$ sama dengan $f(t)$ yang diganti t dengan $-\omega$.

$$\underbrace{\tau \text{sinc}\left(\frac{\tau t}{2}\right)}_{F(t)} \iff \underbrace{2\pi \text{rect}\left(\frac{-\omega}{\tau}\right)}_{2\pi f(-\omega)} = 2\pi \text{rect}\left(\frac{\omega}{\tau}\right)$$

- Karena $\text{rect } x$ adalah fungsi genap, maka $\text{rect}(-x) = \text{rect}(x)$

2. Sifat Penyekala

- Jika

$$f(t) \iff F(\omega)$$

- Maka

$$f(at) \iff \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

3. Sifat Time Shifting

- Jika

$$f(t) \iff F(\omega)$$

- Maka

$$f(t - t_0) \iff F(\omega)e^{-j\omega t_0}$$

Contoh

- Carilah transformasi fourier dari fungsi $e^{-a|t-t_0|}$

Penyelesaian

$$e^{-a|t|} \leftrightarrow \frac{2a}{a^2 + \omega^2}$$

Maka:

$$e^{-a|t-t_0|} \leftrightarrow \frac{2a}{a^2 + \omega^2} e^{-j\omega t_0}$$

4. Sifat Frequency Shifting

– Jika

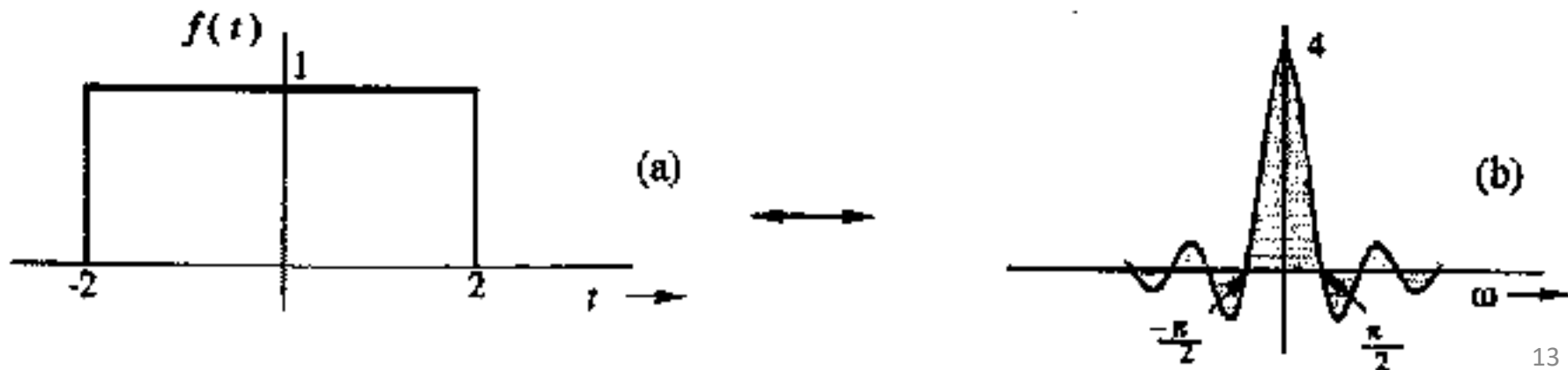
$$f(t) \iff F(\omega)$$

– Maka

$$f(t)e^{j\omega_0 t} \iff F(\omega - \omega_0)$$

• Contoh

- Carilah transformasi fourier dari sinyal termodulasi $f(t) \cdot \cos 10t$. Dimana $f(t)$ adalah pulsa gerbang $\text{rect}(t/4)$ seperti yg digambarkan dibawah ini.



- **Penyelesaian**

- Kita ketahui bahwa:

$$f(t) \cos \omega_0 t = \frac{1}{2} [f(t)e^{j\omega_0 t} + f(t)e^{-j\omega_0 t}]$$

- Berarti:

$$f(t) \cos \omega_0 t \iff \frac{1}{2} [F(\omega - \omega_0) + F(\omega + \omega_0)]$$

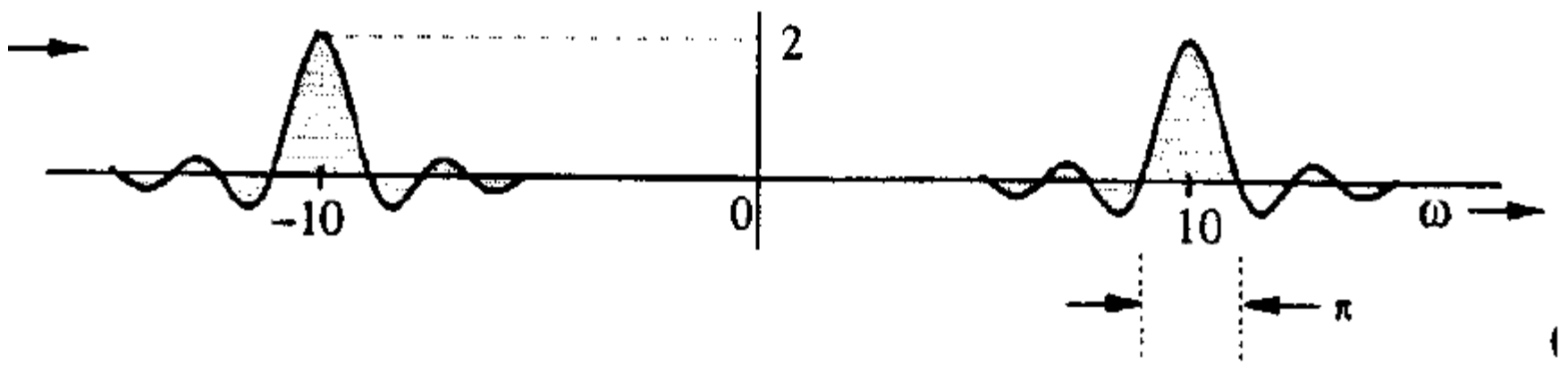
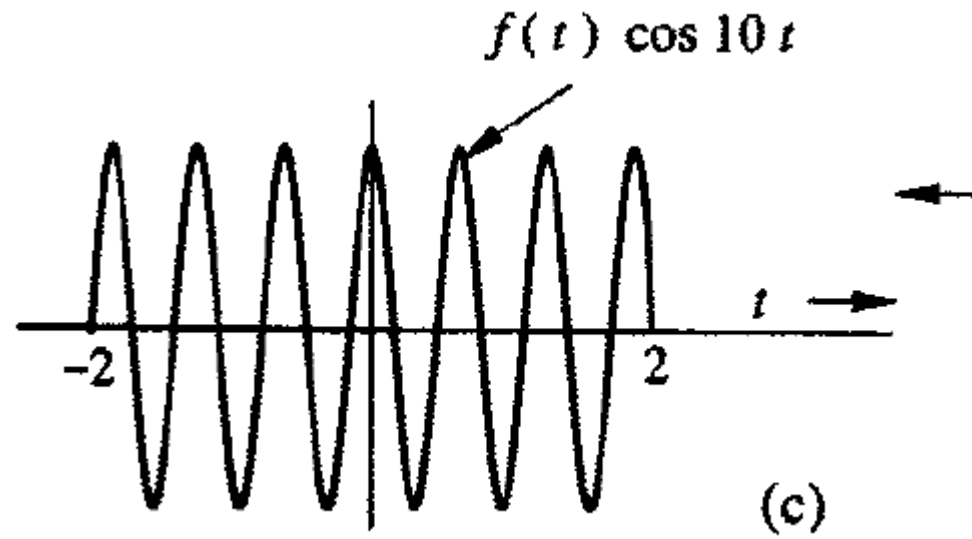
- Sehingga:

$$f(t) \cos 10t \iff \frac{1}{2} [F(\omega + 10) + F(\omega - 10)]$$

- Karena **$f(t) = \text{rect}(t/4)$** , maka **$F(\omega) = 4 \text{sinc}(2\omega)$**

- Sehingga:

$$f(t) \cos 10t \iff 2 \text{sinc}[2(\omega + 10)] + 2 \text{sinc}[2(\omega - 10)]$$



Operasi Transformasi Fourier

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$k f(t)$	$k F(\omega)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Frequency convolution	$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^t f(x) dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$