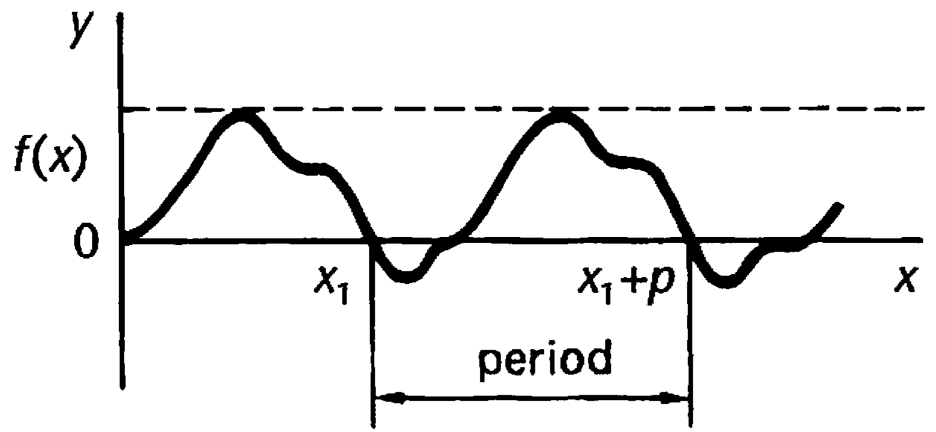
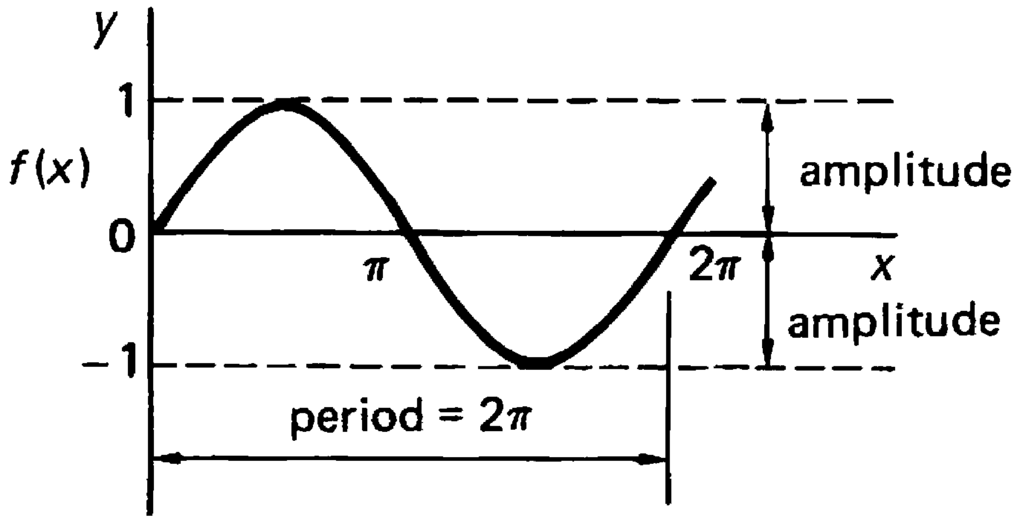
Fungsi Periodik

- Fungsi f(x) dikatakan periodik apabila nilai fungsi tersebut berulang pada interval reguler .
- Interval reguler antara pengulangan adalah periode dari osilasi.

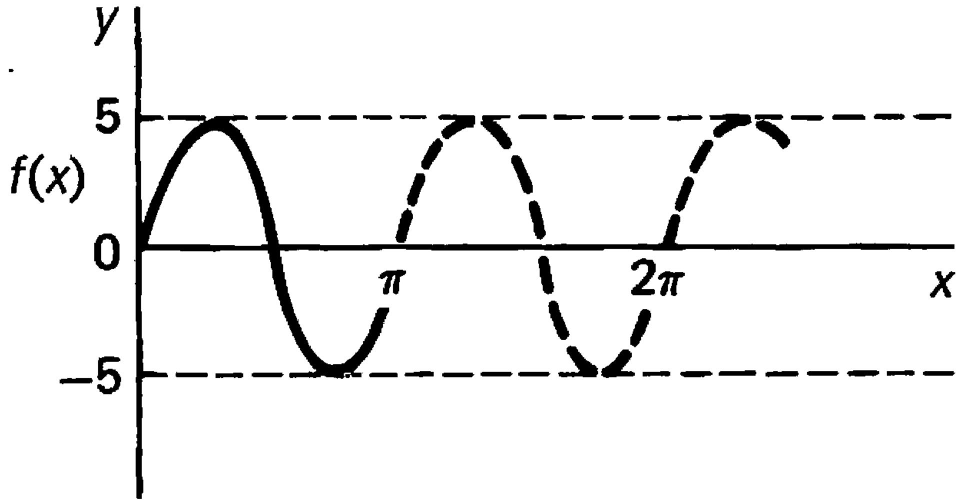


Grafik y = A sin nx

- (a) y = sin x
  - Contoh nyata dari fungsi periodik adalah y = sin x.
  - Periodenya 360^0 atau 2π dengan amplituda maksimum = 1



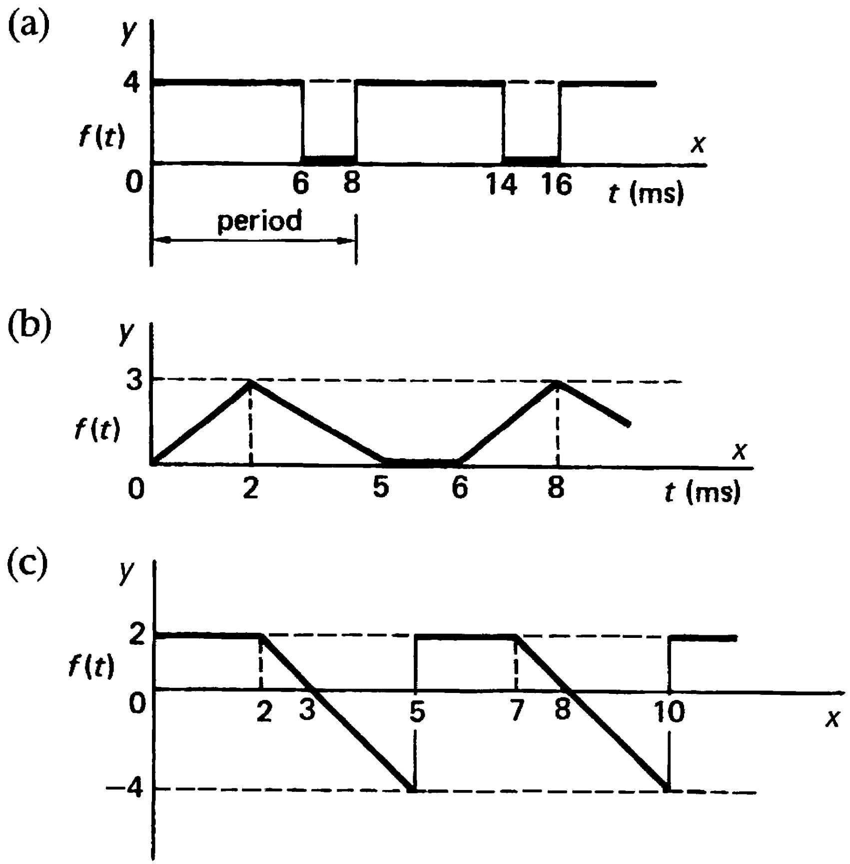
- (b) y = 5 sin 2x
  - Amplitudanya adalah 5.
  - Periodanya adalah 180^0 dan ada 2 gelombang penuh dalam 360^0.



- (c) y = A sin nx
  - Dari dua contoh sebelumnya, maka bisa disimpulkan bahwa fungsi y = A sin nx mempunyai amplituda A dan periode = $360^0/n$ dan ada n buah gelombang penuh dalam 360^0.

Fungsi Periode Bukan Sinus

- Contoh
  - Pada kasus berikut, sumbu x memuat skala dari t dalam mili detik.



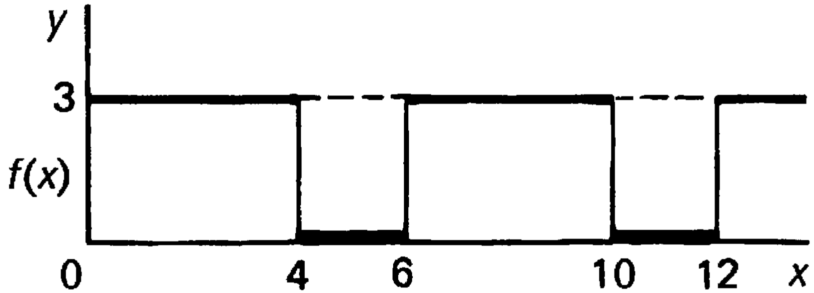
period = 8 ms

period = 6 ms…

period = 5 ms..

Deskripsi Analitik Fungsi Periodik

- Contoh 1



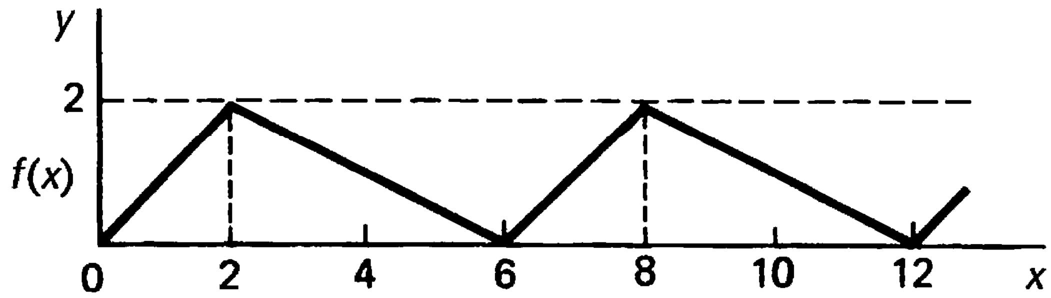
  - Antara x = 0 dan x = 4, y = 3, berarti f(x) = 3 $0 < x < 4$
  - Antara x = 4 dan x = 6, y = 0, berarti f(x) = 0 $4 < x < 6$
  - Jadi kita bisa mendefinisikan fungsi sebagai

$$f(x) = \begin{cases} 3 & 0 < x < 4 \\ 0 & 4 < x < 6 \end{cases}$$

$$f(x+6) = f(x)$$

  - Baris terahir menandakan bahwa fungsi adalah periodik dengan periode 6 satuan.

Example 2



In this case

(a) Between $x = 0$ and $x = 2$, $y = x$ i.e. $f(x) = x \qquad 0 < x < 2$

(b) Between $x = 2$ and $x = 6$, $y = -\dfrac{x}{2} + 3$, i.e. $f(x) = 3 - \dfrac{x}{2} \qquad 2 < x < 6$

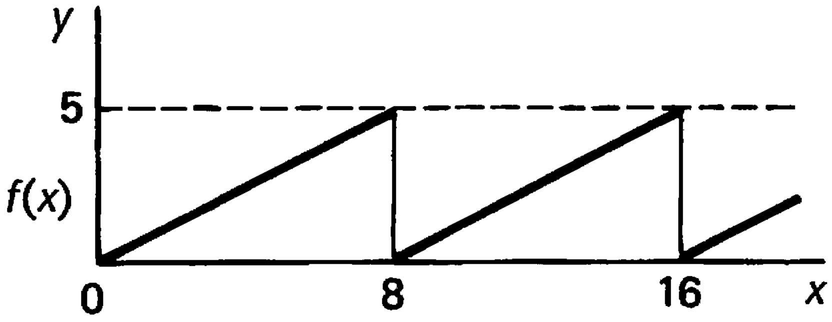
(c) The period is 6 units i.e. $f(x+6) = f(x)$.

So we have

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 3 - \dfrac{x}{2} & 2 < x < 6 \end{cases}$$

$$f(x+6) = f(x).$$

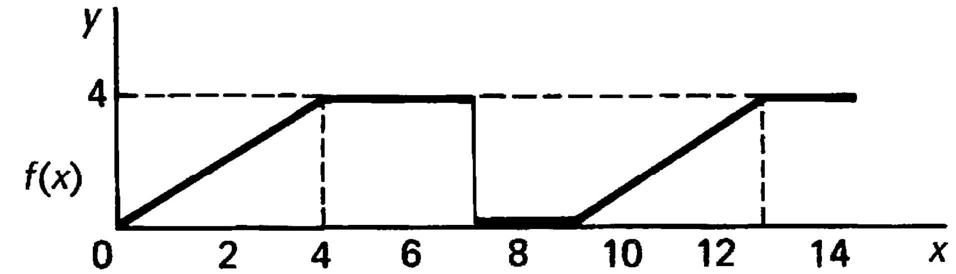
Example 3



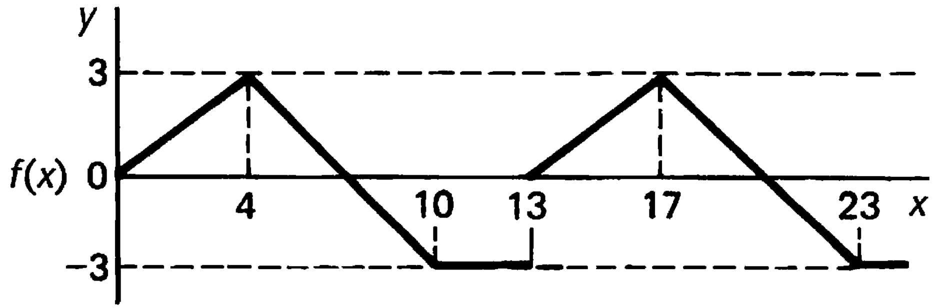
$$f(x) = \frac{5x}{8} \qquad 0 < x < 8$$

$$f(x+8) = f(x)$$

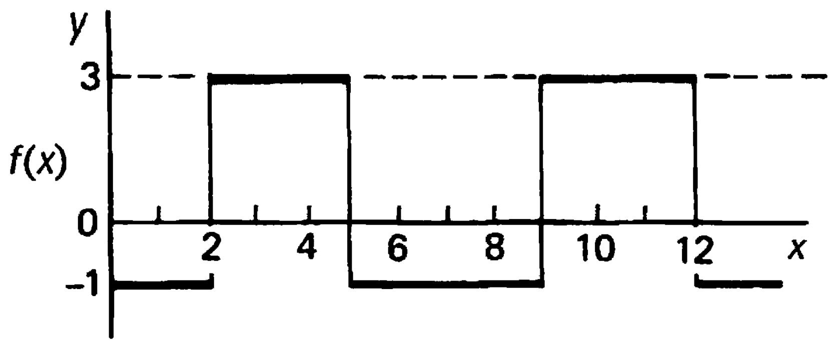
- Latihan
  - Nyatakan fungsi periodik berikut secara analitik
    - (a)



    - (b)



    - (c)



- Jawaban

  - (a) $$f(x) = \begin{cases} x & 0 < x < 4 \\ 4 & 4 < x < 7 \\ 0 & 7 < x < 9 \end{cases}$$

    $$f(x+9) = f(x).$$

  - (b) $$f(x) = \begin{cases} \dfrac{3x}{4} & 0 < x < 4 \\ 7 - x & 4 < x < 10 \\ -3 & 10 < x < 13 \end{cases}$$

    $$f(x+13) = f(x).$$

  - (c) $$f(x) = \begin{cases} -1 & 0 < x < 2 \\ 3 & 2 < x < 5 \\ -1 & 5 < x < 7 \end{cases}$$

    $$f(x+7) = f(x).$$

Integaral Fungsi Periodik

1 $\int_{-\pi}^{\pi} dx = [x]_{-\pi}^{\pi} = 2\pi$

2 $\int_{-\pi}^{\pi} \cos nx \, dx = 0$

3 $\int_{-\pi}^{\pi} \sin nx \, dx = 0$

4 $\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \pi\delta_{mn}$ where $\delta_{mn} = \begin{cases} 1 \text{ if } m = n \\ 0 \text{ if } m \neq n \end{cases}$

(δ_{mn} is called the Kronecker delta)

5 $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \pi\delta_{mn}$

6 $\int_{-\pi}^{\pi} \cos mx \sin nx \, dx = 0$

Deret Fourier

- Jika suatu fungsi f(x) mempunyai periode 2π, maka deret fourier fungsi tersebut adalah:

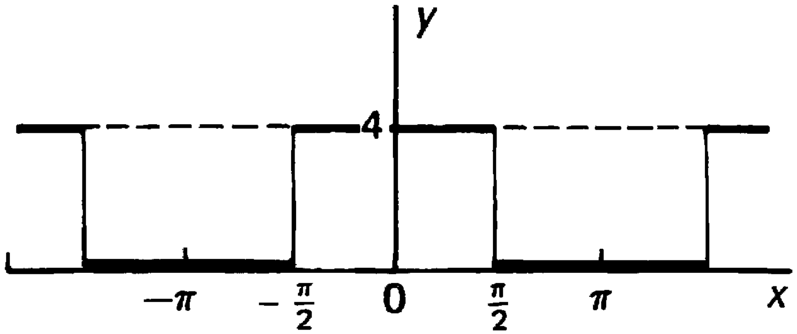
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

- a_n dan b_n adalah konstanta yang disebut koefisien fourier.

- Dimana $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x$, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x \quad (n \neq 0)$, dan

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x.$$

- Contoh 1



Find the Fourier series for the function shown.

Consider one cycle between $x = -\pi$ and $x = \pi$.

The function can be defined by $f(x) = \begin{cases} 0 & -\pi < x < -\dfrac{\pi}{2} \\ 4 & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ 0 & \dfrac{\pi}{2} < x < \pi \end{cases}$

$$f(x + 2\pi) = f(x).$$

(a) As before $f(x) = \dfrac{1}{2}a_0 + \sum_{n=1}^{\infty}\{a_n \cos nx + b_n \sin nx\}$

$$a_0 = \frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2} 0\,\mathrm{d}x + \int_{-\pi/2}^{\pi/2} 4\,\mathrm{d}x + \int_{\pi/2}^{\pi} 0\,\mathrm{d}x\right\}$$

$$= \frac{1}{\pi}\Big[4x\Big]_{-\pi/2}^{\pi/2} \qquad \therefore\ a_0 = 4$$

(b) *To find a_n*

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x$$

$$\therefore\ a_n = \frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2} (0)\cos nx\,\mathrm{d}x + \int_{-\pi/2}^{\pi/2} 4\cos nx\,\mathrm{d}x + \int_{\pi/2}^{\pi} (0)\cos nx\,\mathrm{d}x\right\}$$

$$a_n = \frac{8}{\pi n}\sin\frac{n\pi}{2}$$

If n is even $\qquad a_n = 0$

If $n = 1, 5, 9, \ldots \qquad a_n = \frac{8}{n\pi}$

If $n = 3, 7, 11, \ldots \qquad a_n = -\frac{8}{n\pi}$

(c) *To find b_n*

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x = \ldots\ldots\ldots\ldots$$

$$b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{-\pi/2}(0)\sin nx\,\mathrm{d}x + \int_{-\pi/2}^{\pi/2} 4\sin nx\,\mathrm{d}x + \int_{\pi/2}^{\pi}(0)\sin nx\,\mathrm{d}x\right\}$$

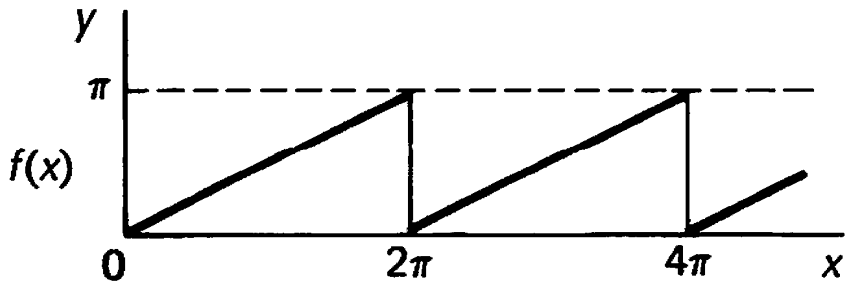
$$= \frac{4}{\pi}\int_{-\pi/2}^{\pi/2}\sin nx\,\mathrm{d}x = \frac{4}{\pi}\left[\frac{-\cos nx}{n}\right]_{-\pi/2}^{\pi/2}$$

$$= -\frac{4}{n\pi}\left\{\cos\frac{n\pi}{2} - \cos\left(\frac{-n\pi}{2}\right)\right\} = 0 \qquad \therefore\ b_n = 0$$

So with $a_0 = 4$; a_n as stated above; $b_n = 0$; the Fourier series is

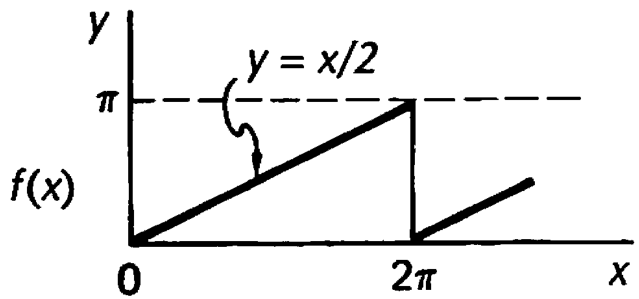
$$f(x) = 2 + \frac{8}{\pi}\left\{\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \frac{1}{7}\cos 7x + \ldots\right\}$$

Example 2



Determine the Fourier series to represent the periodic function shown.

It is more convenient here to take the limits as 0 to 2π.



The function can be defined as

$$f(x) = \frac{x}{2} \qquad 0 < x < 2\pi$$

$$f(x + 2\pi) = f(x) \quad \text{i.e. period} = 2\pi.$$

Now to find the coefficients.

(a) $$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,\mathrm{d}x = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{x}{2}\right)\mathrm{d}x = \frac{1}{4\pi}\left[x^2\right]_0^{2\pi}$$

$$= \pi \qquad\qquad \therefore\ a_0 = \pi$$

(b) $$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,\mathrm{d}x = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{x}{2}\right)\cos nx\,\mathrm{d}x$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\cos nx\,\mathrm{d}x$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} x\cos nx\,\mathrm{d}x = \frac{1}{2\pi}\left\{\left[\frac{x\sin nx}{n}\right]_0^{2\pi} - \frac{1}{n}\int_0^{2\pi}\sin nx\,\mathrm{d}x\right\}$$

$$= \frac{1}{2\pi}\left\{(0-0) - \frac{1}{n}(0)\right\} = 0 \qquad\qquad \therefore\ a_n = 0$$

(c) $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,\mathrm{d}x = \ldots\ldots\ldots\ldots$

$$b_n = -\frac{1}{n}$$

$$\boxed{f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\{a_n\cos nx + b_n\sin nx\}}$$

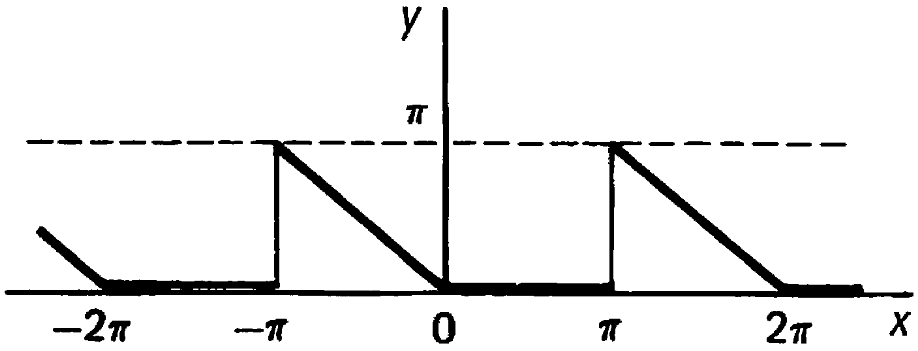
$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\{b_n\sin nx\} \qquad \text{because } a_n = 0$$

$$= \frac{\pi}{2} + \left\{-\frac{1}{1}\sin x - \frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x - \ldots\right\}$$

$$\therefore\ f(x) = \frac{\pi}{2} - \left\{\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots\right\}$$

Example 3

Find the Fourier series for the function defined by



$$f(x) = -x \qquad -\pi < x < 0$$
$$f(x) = 0 \qquad 0 < x < \pi$$
$$f(x + 2\pi) = f(x).$$

The general expressions for a_0, a_n, b_n are

$$a_0 = \ldots\ldots\ldots\ldots$$
$$a_n = \ldots\ldots\ldots\ldots$$
$$b_n = \ldots\ldots\ldots\ldots$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\,\mathrm{d}x = \frac{1}{\pi}\left[-\frac{x^2}{2}\right]_{-\pi}^{0} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\cos nx\,\mathrm{d}x$$

$$= -\frac{1}{\pi}\int_{-\pi}^{0} x\cos nx\,\mathrm{d}x$$

$$= -\frac{1}{\pi}\left\{\left[x\frac{\sin nx}{n}\right]_{-\pi}^{0} - \frac{1}{n}\int_{-\pi}^{0}\sin nx\,\mathrm{d}x\right\}$$

$$= -\frac{1}{\pi}\left\{(0-0) - \frac{1}{n}\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0}\right\} = -\frac{1}{\pi n^2}\{1-\cos n\pi\}$$

But $\cos n\pi = 1$ (n even) or -1 (n odd)

$$\therefore\ a_n = -\frac{2}{\pi n^2}\quad (n \text{ odd})\quad \text{or } 0\quad (n \text{ even})$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\sin nx\,dx$$

$$= -\frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx$$

$$= -\frac{1}{\pi}\left\{\left[x\left(\frac{-\cos nx}{n}\right)\right]_{-\pi}^{0} + \frac{1}{n}\int_{-\pi}^{0}\cos nx\,dx\right\}$$

$$= -\frac{1}{\pi}\left\{\frac{\pi\cos n\pi}{n} + \frac{1}{n}\left[\frac{\sin nx}{n}\right]_{-\pi}^{0}\right\} = -\frac{\cos n\pi}{n}$$

$$\therefore\ b_n = -\frac{1}{n}\quad (n \text{ even});\quad \frac{1}{n}\quad (n \text{ odd})$$

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \dots\right) + \left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + \dots\right)$$

Deret Fourier Fungsi Dengan Perioda T

- Jika suatu fungsi mempunyai perioda T, maka deret fourier fungsi tersebut adalah:

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\{a_n \cos\ n\omega t + b_n \sin\ n\omega t\}$$

$$a_0 = \frac{2}{T}\int_0^T f(t)\,\mathrm{d}t \qquad = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)\,\mathrm{d}t$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,\mathrm{d}t \qquad = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)\cos n\omega t\,\mathrm{d}t$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,\mathrm{d}t \qquad = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)\sin n\omega t\,\mathrm{d}t$$

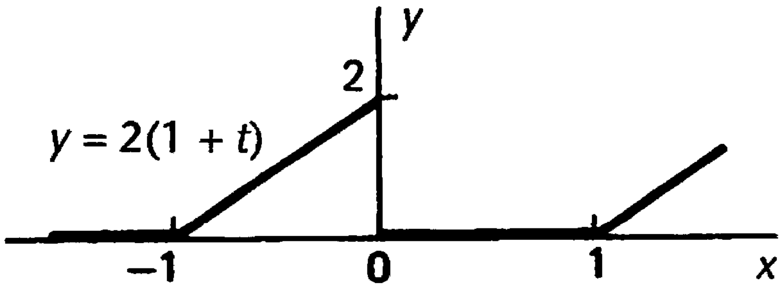
where $\omega = \frac{2\pi}{T}$ i.e. $T = \frac{2\pi}{\omega}$.

- Contoh 1
  - Tentukanlah deret fourier dari fungsi berikut:

$$f(t) = \begin{cases} 2(1+t) & -1 < t < 0 \\ 0 & 0 < t < 1 \end{cases}$$

$$f(t+2) = f(t)$$

- Penyelesaiaan
  - Pertama kita gambarkan bentuk gelombang persamaan:



  - Kita peroleh:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right\}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \{ a_n \cos n\pi t + b_n \sin n\pi t \} \quad \text{because } T = 2$$

– Maka;

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\,\mathrm{d}t = \int_{-1}^{1} f(t)\,\mathrm{d}t = \int_{-1}^{0} 2(1+t)\,\mathrm{d}t + \int_{0}^{1} (0)\,\mathrm{d}t$$

$$= \left[2t + t^2\right]_{-1}^{0} = 1$$

– dan

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\pi t\,\mathrm{d}t = \int_{-1}^{1} f(t)\cos n\pi t\,\mathrm{d}t$$

$$a_n = \int_{-1}^{0} 2(1+t)\cos n\pi t\,\mathrm{d}t$$

$$= 2\left\{\left[(1+t)\frac{\sin\ n\pi t}{n\pi}\right]_{-1}^{0} - \frac{1}{n\pi}\int_{-1}^{0} \sin\ n\pi t\,\mathrm{d}t\right\}$$

$$= 2\left\{(0-0) - \frac{1}{n\pi}\left[-\frac{\cos n\pi t}{n\pi}\right]_{-1}^{0}\right\} = \frac{2}{n^2\pi^2}(1-\cos n\pi)$$

$$= \frac{2}{n^2\pi^2}\left(1-(-1)^n\right)$$

- sehingga;

$$a_n = 0 \quad (n \text{ even}), \quad a_n = \frac{4}{n^2\pi^2} \quad (n \text{ odd})$$

- Berikutnya adalah b_n

$$b_n = \int_{-1}^{0} 2(1+t)\sin n\pi t \,dt$$

$$= 2\left\{\left[(1+t)\frac{-\cos n\pi t}{n\pi}\right]_{-1}^{0} + \frac{1}{n\pi}\int_{-1}^{0}\cos n\pi t\,dt\right\}$$

$$= 2\left\{-\frac{1}{n\pi} + \left[\frac{\sin n\pi t}{n\pi}\right]_{-1}^{0}\right\} = -\frac{2}{n\pi} + \frac{2}{n^2\pi^2}(\sin n\pi) = -\frac{2}{n\pi}$$

- Maka diperolehlah deret fourier dari fungsi adalah:

$$f(t) = \frac{1}{2} + \frac{4}{\pi^2}\left\{\cos \pi t + \frac{1}{9}\cos 3\pi t + \frac{1}{25}\cos 5\pi t + \ldots\right\}$$

$$- \frac{2}{\pi}\left\{\sin \pi t + \frac{1}{2}\sin 2\pi t + \frac{1}{3}\sin 3\pi t + \frac{1}{4}\sin 4\pi t + \ldots\right\}$$

Latihan

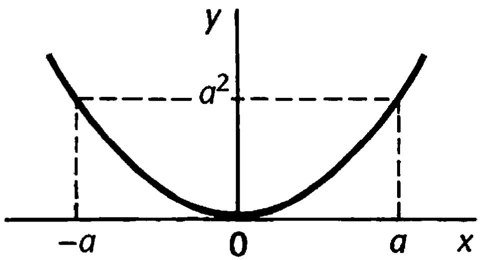
- Tentukanlah deret fourier dari fungsi periodik berikut:

$$f(t) = \begin{cases} 0 & -2 < t < 0 \\ t & 0 < t < 2 \end{cases}$$

$$f(t+4) = f(t).$$

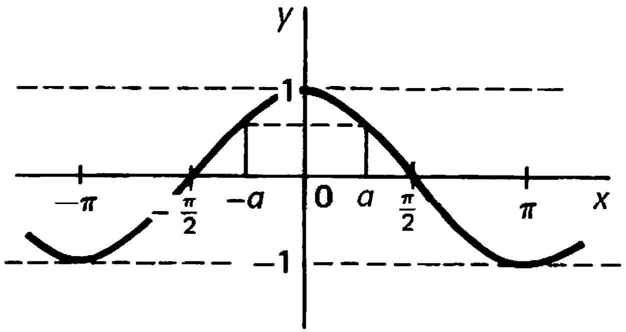
Fungsi genap dan fungsi ganjil

- Fungsi Genap (even)
  - Suatu fungsi f(x) dikatakan genap jika f(-x) = f(x)
  - Contoh



$y = f(x) = x^2$ is an even function because

$$f(-2) = 4 = f(2)$$
$$f(-3) = 9 = f(3) \quad \text{etc.}$$

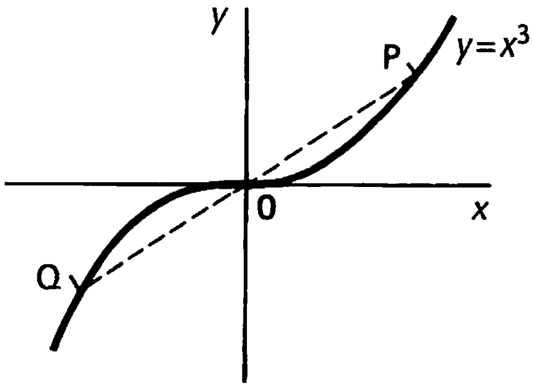


$y = f(x) = \cos x$ is an even function because

$$\cos(-x) = \cos x$$
$$f(-a) = \cos a = f(a).$$

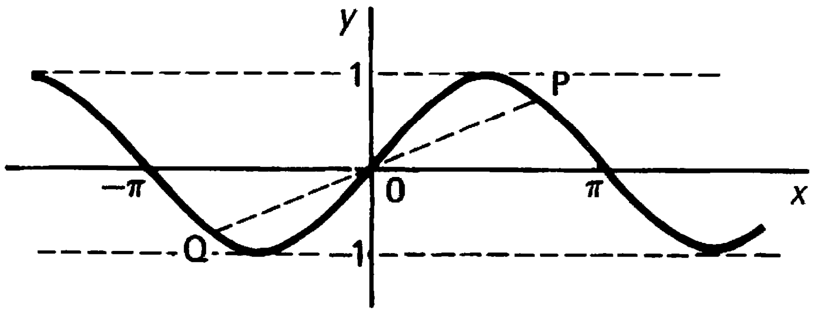
  - Grafik fungsi genap simetris terhadap sumbu y

- Fungsi ganjil (odd)
  - Suatu fungsi dikatakan ganjil apabila f(-x) = -f(x)
  - Contoh



$y = f(x) = x^3$ is an odd function because

$$f(-2) = -8 = -f(2)$$
$$f(-5) = -125 = -f(5) \quad \text{etc.}$$

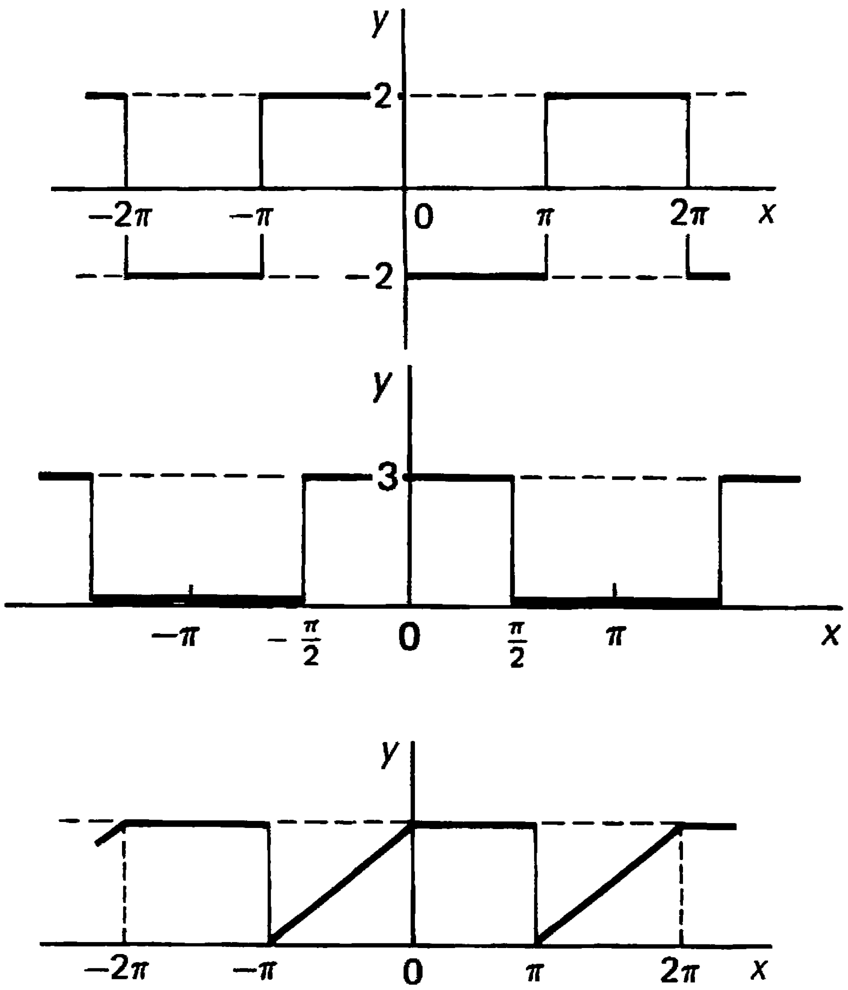


$y = f(x) = \sin x$ is an odd function because

$$\sin(-x) = -\sin x$$
$$f(-a) = -f(a).$$

  - Grafik fungsi ganjil simetris terhadap titik asal (origin)

- Contoh
  - Tentukanlah, apakah fungsi berikut ini genap atau ganji?



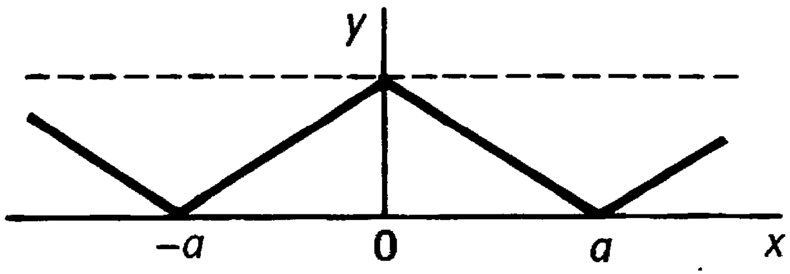
Hasi Kali Fungsi Genap dan Fungsi Ganjil

The rules closely resemble the elementary rules of signs.

$$
\begin{aligned}
(\text{even}) \times (\text{even}) &= (\text{even}) \quad \text{like} \quad & (+) \times (+) &= (+) \\
(\text{odd}) \times (\text{odd}) &= (\text{even}) & (-) \times (-) &= (+) \\
(\text{odd}) \times (\text{even}) &= (\text{odd}) & (-) \times (+) &= (-).
\end{aligned}
$$

Dua hal penting tentang fungsi genap dan fungsi ganjil;

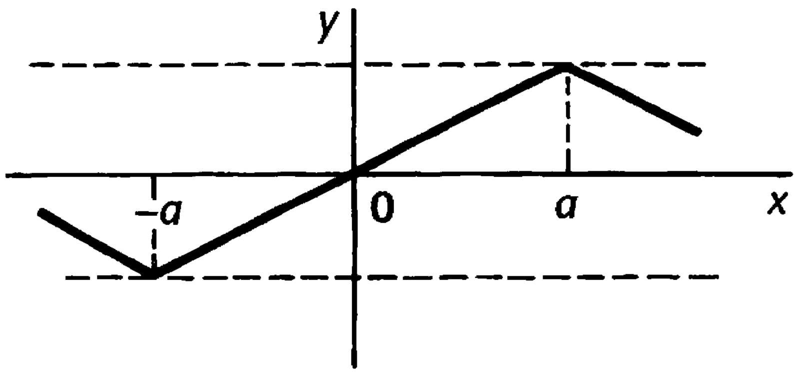
(a)



For an *even* function

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

(b)



For an *odd* function

$$\int_{-a}^{a} f(x)dx = 0$$

- Teorema I
  - Jika suatu fungsi adalah genap, maka deret fourier fungsi tersebut adalah:

$$\therefore \quad f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

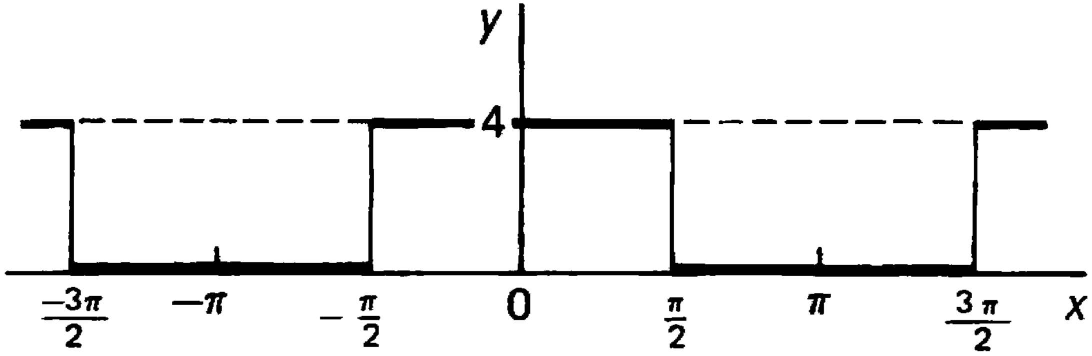
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} f(x)\,\mathrm{d}x \qquad \therefore \quad a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,\mathrm{d}x$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,\mathrm{d}x.$$

  - Sedangkan suku sinus bernilai nol; genap x ganjil = ganjil.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x = 0. \quad \therefore \quad b_n = 0$$

- Contoh
  - Tentukanlah deret fourier dari fungsi berikut:



- Penyelesaian
  - Karena grafik diatas adalah grafik fungsi genap, maka deret fouriernya adalah

$$\therefore \quad f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} f(x)\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi/2} 4\,\mathrm{d}x = \frac{2}{\pi}\Big[4x\Big]_0^{\pi/2} = 4$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,\mathrm{d}x$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,\mathrm{d}x = \frac{2}{\pi}\int_0^{\pi/2} 4\cos nx\,\mathrm{d}x$$

$$= \frac{8}{\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi/2} = \frac{8}{\pi n}\sin\frac{n\pi}{2}$$

But $\sin\frac{n\pi}{2} = 0$ for n even

$= 1$ for $n = 1, 5, 9, \ldots$

$= -1$ for $n = 3, 7, 11, \ldots$ Hence the result stated.

$$\boxed{f(x) = 2 + \frac{8}{\pi}\left\{\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \frac{1}{7}\cos 7x + \ldots\right\}}$$

- Teorema II
  - Jika suatu fungsi adalah ganjil, maka deret fourier fungsi tersebut hanya mengandung komponen sinus.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin\, nx \, dx$$

  - Suku awal dan suku cosinus bernilai nol;

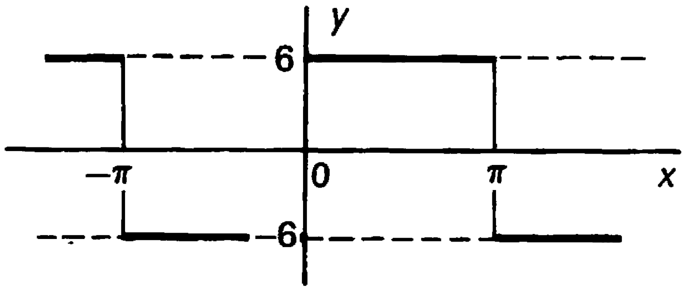
$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx. \qquad \text{But } f(x) \text{ is odd} \qquad \therefore\ a_0 = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} (\text{odd function})\, dx = 0$$

$$a_n = 0 \quad (\text{including } a_0 = 0)$$

- Contoh
  - Tentukanlah deret fourier dari fungsi berikut:



$$f(x) = -6 \qquad -\pi < x < 0$$
$$f(x) = 6 \qquad 0 < x < \pi$$
$$f(x + 2\pi) = f(x).$$

- Penyelesaian
  - Karena grafik diatas adalah grafik fungsi ganjil, maka deret fouriernya adalah $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

$$b_n = \frac{2}{\pi}\int_0^{\pi} 6 \sin nx \, dx = \frac{12}{\pi}\left[\frac{-\cos nx}{n}\right]_0^{\pi} = \frac{12}{\pi n}(1 - \cos n\pi).$$

$$f(x) = \frac{24}{\pi}\left\{\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right\}$$

Quiz....

- Tentukan deret fourier dari grafik berikut: